I. ABSTRACT

In this article, we propose a link-based mixed integer linear programming formulation (MILP) for solving routing and spectrum assignment (RSA) problem for elastic optical networks (EON). This formulation implements physical impairments, signal regeneration, wavelength conversion and multiple modulation schemes.

II. MILP FORMULATION

Objective function:

\[ \min c \]  

The objective function is to minimize the highest frequency index required by the network traffic. This is because it measures the spectrum resource required to guarantee accommodate all traffic.
### TABLE I
**SETS USED BY ILP**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Set of nodes in the network.</td>
</tr>
<tr>
<td>$\langle N \rangle$</td>
<td>Number of nodes in the network.</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of unidirectional links in the network. Each link $L_{ij}$ is represented by its source and destination node, $L_{ij} \in L$.</td>
</tr>
<tr>
<td>$\langle L \rangle$</td>
<td>Number of unidirectional links in the network.</td>
</tr>
<tr>
<td>$D$</td>
<td>Set of unidirectional traffic demands. Each demand $D_{sd}$ is represented by its source and destination node, $D_{sd} \in D$.</td>
</tr>
<tr>
<td>$\langle D \rangle$</td>
<td>Number of unidirectional demands in the network.</td>
</tr>
</tbody>
</table>

### TABLE II
**PARAMETERS USED BY ILP**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Br_{sd}$</td>
<td>Bit rate requested by demand $D_{sd}$.</td>
</tr>
<tr>
<td>$Sei$</td>
<td>Inverse spectrum efficiency according to particular modulation scheme (e.g., 0.5 for QPSK).</td>
</tr>
<tr>
<td>$Length_{ij}$</td>
<td>Length of link $L_{ij}$ in kms.</td>
</tr>
<tr>
<td>$S_{n,sd}$</td>
<td>Relationship between nodes and demands: $S_{n,sd} = -1$ if node $n$ is the source node of demand $D_{sd}$ (i.e., $n = s$); $S_{n,sd} = 1$ if node $n$ is the destination node of demand $D_{sd}$ (i.e., $n = d$); $S_{n,sd} = 0$ otherwise (i.e., $n \neq s$, $n \neq d$).</td>
</tr>
<tr>
<td>$T$</td>
<td>Total spectrum required by the network traffic: $T = \sum_{D_{sd} \in D} Br_{sd} \times Sei$.</td>
</tr>
<tr>
<td>$G$</td>
<td>Spectrum required by guard band.</td>
</tr>
</tbody>
</table>

### TABLE III
**VARIABLES USED BY ILP**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{sd}$</td>
<td>Starting frequency index of demand $D_{sd}$.</td>
</tr>
<tr>
<td>$V_{ij,sd}$</td>
<td>Link assignment: $V_{ij,sd} = 1$ if link $L_{ij}$ is assigned to demand $D_{sd}$; $V_{ij,sd} = 0$ otherwise.</td>
</tr>
<tr>
<td>$\delta_{sd,s'd'}$</td>
<td>Order of the starting frequency index of demand $D_{sd}$ and $D_{s'd'}$: $\delta_{sd,s'd'} = 1$ if $F_{sd} \leq F_{s'd'}$, $\delta_{sd,s'd'} = 0$ if $F_{sd} &gt; F_{s'd'}$.</td>
</tr>
<tr>
<td>$c$</td>
<td>Highest frequency index required by the network traffic.</td>
</tr>
</tbody>
</table>

### Constraints:

Highest frequency index:

$$c \geq F_{sd} + Bw_{sd} \quad \forall D_{sd} \in D$$

(2)

where $Bw_{sd}$ is the bandwidth required by $D_{sd}$. $Bw_{sd} = Br_{sd} \times Sei$.

\(^{1}\)This relationship between two demands is only of interest if they share a link. We use this relationship in the following constraints to guarantee no overlapping between spectrum assigned to multiple demands.
Conservation flow constraint:

\[
\sum_{L_{ij} \in L, j = n} V_{ij, sd} - \sum_{L_{ij} \in L, i = n} V_{ij, sd} = S_{n, sd} \quad \forall n \in N, D_{sd} \in D
\]  

No spectrum overlapping, \( \forall D_{sd}, D_{s'd'} \in D, L_{ij} \in L \):

\[
\delta_{sd, s'd'} + \delta_{s'd', sd} = 1
\]

\[
F_{sd} - F_{s'd'} \leq T \times (1 - \delta_{sd, s'd'} + 2 - V_{ij, sd} - V_{ij, s'd'})
\]

\[
F_{sd} - F_{s'd'} + Bw_{sd} + G \leq (T + G) \times (1 - \delta_{sd, s'd'} + 2 - V_{ij, sd} - V_{ij, s'd'})
\]

**Additional variables and constraints to implement transmission reach constraint:**

**TABLE IV**

<table>
<thead>
<tr>
<th>Parameters used by transmission reach constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Tr )</td>
</tr>
<tr>
<td>( N_r )</td>
</tr>
</tbody>
</table>

Transmission reach formula according to particular bit rate and spectrum efficiency:

\[
Tr = \frac{\alpha}{Br_{sd}} + \beta \times Sei + \gamma
\]

where, \( \alpha, \beta, \gamma \) are coefficients derived from polynomial curve fitting based on experimental data recorded in [1].

**TABLE V**

<table>
<thead>
<tr>
<th>Variables used by transmission reach constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{ij, sd} )</td>
</tr>
<tr>
<td>( Y_{n, sd} )</td>
</tr>
</tbody>
</table>

Objective for transmission reach:

\[
\min a_1 \times c + a_2 \times \sum_{n \in N} Rn_n + a_3 \times \sum_{n \in N} Rn_{n,c}
\]

where \( a_1, a_2, a_3 \) are coefficients representing cost function for each resource. This objective minimizes
the total cost of spectrum usage and regeneration resource usage according to their cost function. While we don’t have exact cost relationship among the three, but network designers can base their objective function on realistic requirement.

**Constraints for transmission reach:**

Transmission reach constraints with pre-selected regeneration nodes, \( \forall D_{sd} \in D, L_{ij} \in L \):

\[
U_{ij,sd} \leq V_{ij,sd} \times Tr,
\]

\[
U_{ij,sd} \leq Y_{i,sd}
\]

\[
Y_{i,sd} - U_{ij,sd} \leq Tr \times (1 - V_{ij,ld})
\]

\[
Y_{n,sd} = 0, \quad \forall n \in N^r \text{ or } n = s
\]

\[
Y_{n,sd} = \sum_{L_{ij} \in L: j = n} U_{ij,sd} + \text{Length}_{ij} \times V_{ij,sd}, \quad \forall n \notin N^r \text{ and } n \neq s
\]

If regeneration nodes are not pre-selected, we treat them as binary variables \( R_{n} \) and re-write the constraints above:

**TABLE VI**

**VARIABLES USED BY TRANSMISSION REACH CONSTRAINT**

| \( R_{n} \) | Regeneration nodes: \( R_{n} = 1 \) if node \( n \) is equipped with regeneration circuit; \( R_{n} = 0 \) otherwise. |
| \( R_{n,c} \) | Number of regeneration circuits equipped on node \( n \). |
| \( R_{n,sd} \) | Regeneration at node \( n \): \( R_{n,sd} = 1 \) if demand \( D_{sd} \) is regenerated as node \( n \), \( R_{n,sd} = 0 \) otherwise. For \( R_{n,sd} = 1 \), node \( n \) has to be a regeneration node, i.e., \( R_{n} = 1 \), but also its regeneration circuit has to be used by demand \( D_{sd} \). |
| \( X_{ij,sd} \) | The distance used to calculate \( Y_{n,sd} \) based on choice of regeneration at node \( i \). \( X_{ij,sd} = U_{ij,sd} \) if \( R_{n,sd} = 1 \), \( X_{ij,sd} = 0 \), otherwise. |

Transmission reach constraints without pre-selected regeneration nodes, \( \forall D_{sd} \in D, L_{ij} \in L \):

\[
U_{ij,sd} \leq V_{ij,sd} \times Tr,
\]

\[
U_{ij,sd} \leq Y_{i,sd}
\]

\[
Y_{i,sd} - U_{ij,sd} \leq Tr \times (1 - V_{ij,ld})
\]

\[
Y_{n,sd} \leq 0
\]

\[
Y_{n,sd} = \sum_{L_{ij} \in L: j = n} X_{ij,sd} + \text{Length}_{ij} \times V_{ij,sd},
\]
Regeneration circuit usage,

\[ R_{n,c} = \sum_{D_{sd} \in D} R_{n,sd} \tag{19} \]

Regeneration node assignment,

\[ R_n \times R_{n,c,\text{MAX}} \geq R_{n,c} \tag{20} \]

where \( R_{n,c,\text{MAX}} \) is the largest number of regeneration circuit allowed to be equipped on a regeneration node.

**Additional variables and constraints to implement wavelength conversion:**

<table>
<thead>
<tr>
<th>TABLE VII</th>
<th>VARIABLES USED BY WAVELENGTH CONVERSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{ij,sd} )</td>
<td>Starting frequency index of demand ( D_{sd} ) on link ( L_{ij} ).</td>
</tr>
</tbody>
</table>

**Constraints for wavelength conversion:**

Wavelength continuity constraint, \( \forall n \in N \)

\[
\sum_{L_{ij} \in L : j = n} F_{ij,sd} - \sum_{L_{ij} \in L : i = n} F_{ij,sd} \geq -T \times (R_{n,sd} + |S_{n,sd}|) \tag{21}
\]

\[
\sum_{L_{ij} \in L : j = n} F_{ij,sd} - \sum_{L_{ij} \in L : i = n} F_{ij,sd} \leq T \times (R_{n,sd} + |S_{n,sd}|) \tag{22}
\]

This constraint requires that if node \( n \) is an intermediate node for demand \( D_{sd} \), i.e., \( n \neq s, n \neq d \) and \( n \) is not used as regeneration node for demand \( D_{sd} \), then the sum of the starting frequency assignment coming into node \( n \) equals the sum of the starting frequency assignment going out of node \( n \). For other cases, this constraint doesn’t do anything.

**Additional variables and constraints to implement multiple modulation schemes:** When each demand has different spectral efficiency, their transmission reach also varies. This is implemented by replacing \( S_e \) in the transmission reach formula Eq. (7) with \( S_{ei,sd} \). \( S_{ei,sd} \) is also used to replace \( S_e \) in the bandwidth formula.
Additional variables and constraints to implement modulation conversion:

| TABLE IX |
| Variables used by modulation conversion |

\[ Se_{ij, sd} \mid \text{Inverse spectral efficiency of demand } D_{sd} \text{ on link } L_{ij}. \]

Constraints for modulation conversion:

Modulation continuity constraint,

\[
\sum_{L_{ij} \in L: j=n} Se_{ij, sd} - \sum_{L_{ij} \in L: i=n} Se_{ij, sd} \geq -Se_{\text{MAX}} \times (R_{n,sd} + |S_{n,sd}|) \tag{23}
\]

\[
\sum_{L_{ij} \in L: j=n} Se_{ij, sd} - \sum_{L_{ij} \in L: i=n} Se_{ij, sd} \leq Se_{\text{MAX}} \times (R_{n,sd} + |S_{n,sd}|) \tag{24}
\]

Similar to the wavelength continuity constraint, this constraint requires that spectral efficiency can only be converted at the node where demand is regenerated, i.e., \( R_{n,sd} = 1 \).

References