Adaptive Control Technologies for Aerospace Systems

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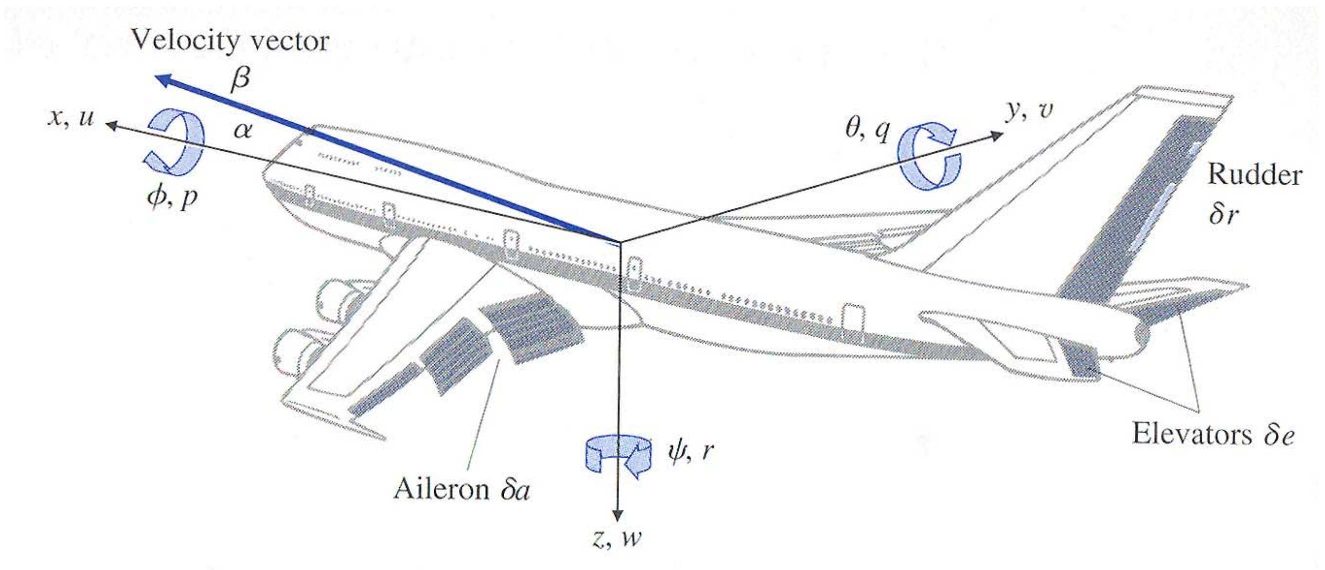
Part I: Adaptive Control Theory

- Issues in Automatic Feedback Control
- Adaptive Control Methodology
- Direct Model Reference Adaptive Control
- Indirect Adaptive Pole Placement Control
- Multivariable Adaptive Control
- Nonlinear Adaptive Control
- Performance, Convergence and Robustness

Aircraft Flight Control System Models



- State variables

x, y, z = position coordinates
u, v, w = velocity coordinates
p = roll rate
q = pitch rate
r = yaw rate

ϕ = roll angle
θ = pitch angle
ψ = yaw angle
β = side-slip angle
α = angle of attack

- Nonlinear equations of motion (in body axis)
  - Force equations:

$$\begin{aligned} m(\dot{u}+qw-rv) &= X-mg\sin\theta+T\cos\varepsilon \\ m(\dot{v}+ru-pw) &= Y+mg\cos\theta\sin\phi \\ m(\dot{w}+pv-qu) &= Z+mg\cos\theta\cos\phi-T\sin\varepsilon \end{aligned}$$

T: engine thrust; κ: thrust angle; X, Y, Z: aerodynamic forces

  - Moment equations:

$$\begin{aligned} I_x\dot{p}+I_{xz}\dot{r}+(I_z-I_y)qr+I_{xz}qp &= L \\ I_y\dot{q}+(I_x-I_z)pr+I_{xz}(r^2-p^2) &= M \\ I_z\dot{r}+I_{xz}\dot{p}+(I_y-I_x)qp-I_{xz}qr &= N \end{aligned}$$

L, M, N: aerodynamic torques

- Linearized longitudinal equations

$$\begin{bmatrix}\dot{u}\\ \dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} X_u & X_w & -W_0 & -g_0\cos\theta_0 \\ Z_u & Z_w & U_0 & -g_0\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u\\ w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix} X_{\delta e}\\ Z_{\delta e}\\ M_{\delta e}\\ 0\end{bmatrix}\delta e$$

output = θ: pitch angle perturbation

- Linearized lateral equations

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\\ \dot{p}\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix} Y_v & -U_0 & V_0 & g_0\cos\theta_0 \\ N_v & N_r & N_p & 0 \\ L_v & L_r & L_p & 0 \\ 0 & \tan\theta_0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta\\ r\\ p\\ \phi\end{bmatrix} + \begin{bmatrix} Y_{\delta r} & Y_{\delta a}\\ N_{\delta r} & N_{\delta a}\\ L_{\delta r} & L_{\delta a}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta r\\ \delta a\end{bmatrix}$$

output = r: yaw rate perturbation

Control System Dynamic Models

- Nonlinear models

$$\dot{x} = f(x,u,w),\ y = h(x,u,v)$$

state vector $x \in R^n$, input u, output y, disturbances w, v; or

$$\dot{x} = f(x) + g(x)u + d(x)w,\ y = h(x,u) + v$$

- Linear state-variable model

$$\dot{x} = Ax + Bu + B_w w,\ y = Cx + Du + v$$

- Linear time-invariant input-output model

$$\begin{aligned} y(t) &= G(s)[u](t) + d(t) \\ G(s) &= G_0(s)(1 + \mu\Delta_m(s)) + \mu\Delta_a(s),\ G_0(s) = k_p \frac{Z(s)}{P(s)} \end{aligned}$$

$\mu\Delta_a(s)$, $\mu\Delta_m(s)$: additive, multiplicative unmodeled dynamics.

Issues of Automatic Feedback Control

- System modeling
- Control objectives

  stability, transient, tracking, optimality, robustness
- Parametric uncertainties

  payload variation, component aging, condition change
- Structural uncertainties

  component failure, unmodeled dynamics
- Environmental uncertainties

  external disturbances
- Nonlinearities

  smooth functions and nonsmooth (“hard”) characteristics

Adaptive Control Methodology

- Adapting to parametric uncertainties
- Robust to structural and environmental uncertainties
- Aimed at both stability (signal boundedness) and tracking
- Self-tuning of controller parameters
- Systematic design and analysis
- Real-time implementable
- Effective for failures and nonsmooth nonlinearities
- High potential for applications
- Attractive open and challenging issues

Adaptive Control versus Fixed Control

- System

$$\dot{y}(t) = (a_p + \Delta)y(t) + u(t)$$

- Reference model

$$\dot{y}_r(t) = -a_r y_r(t) + r(t),\ a_r > 0$$

- Ideal controller for $\Delta = 0$

$$u(t) = k^* y(t) + r(t),\ k^* = -a_p - a_r$$

- Ideal performance for $\Delta = 0$

$$\dot{y}(t) = -a_r y(t) + r(t),\ \lim_{t \to \infty}(y(t) - y_r(t)) = 0$$

- Fixed controller for $\Delta \in [\Delta_1, \Delta_2]$

$$u(t) = ky(t) + r(t),\ k < -a_p - \Delta_2$$

- Closed-loop system

$$\dot{y}(t) = -a_r y(t) + (a_p + \Delta + k + a_r) y(t) + r(t),$$

$$e(t) = y(t) - y_r(t) = \frac{a_p + \Delta + k + a_r}{s + a_r} \, \frac{1}{s - a_p - \Delta - k}[r](t)$$

- Tracking performance (for $r(t) = 1$)

$$e_{ss} = \lim_{t \to \infty} e(t) = -\frac{a_p + \Delta + k + a_r}{a_r(a_p + \Delta + k)}$$

- Adaptive controller

$$u(t) = k(t) y(t) + r(t)$$

$$\dot{k}(t) = -\gamma e(t) y(t), \; \gamma > 0$$

with $k(0)$ being arbitrary, leading to $\lim_{t\to\infty} e(t) = 0$.

- Observation: an adaptive controller ensures desired stability and tracking, despite any large parameter uncertainty Δ.

Brief History of Adaptive Control

- Before 1960s

  ideas of adaptive control for dynamic processes
- 1960s - 1970s

  state feedback designs based on Lyapunov methods
- 1970s - 1980s

  output feedback designs based on passivity and estimation errors
- 1980s

  robust adaptive control, multivariable adaptive control
- 1990s - present

  adaptive nonlinear control, neural network based designs

  adaptive switching designs, multiple-model based designs

  adaptive inverse compensation, adaptive learning, etc.

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  Adaptive control and parameter estimation theory

- Krstić, M., I. Kanellakopoulos and P. V. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, New York, 1995.

  Adaptive nonlinear backstepping control theory

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  Adaptive linear and nonlinear control theory

Model Reference Adaptive Control

System model

$$\dot{x}(t) = Ax(t) + Bu(t),\ x(t) \in R^n,\ y(t) = Cx(t)$$

$A \in R^{n\times n}$, $B \in R^{n\times 1}$, $C \in R^{1\times n}$: unknown parameter matrices

Control objective

Design a feedback control $u(t)$ to ensure

(i) closed-loop signal boundedness, and

(ii) $x(t)$ or $y(t)$ tracking a reference signal $x_r(t)$ or $y_r(t)$

Reference system

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t),\ x_r(t) \in R^n,\ y_r(t) = C_r x_r(t)$$

$A_r \in R^{n\times n}$stable, $B_r \in R^{n\times 1}$, $C_r \in R^{1\times n}$: known; $r(t)$: bounded

State Feedback Design for State Tracking

Ideal controller for known system

$$u(t) = K_1^{*T} x(t) + k_2^* r(t)$$

Matching conditions

$$A + BK_1^{*T} = A_r,\ Bk_2^* = B_r$$

Closed-loop system

$$\dot{x}(t) = A_r x(t) + B_r r(t)$$

the tracking error $e(t) = x(t) - x_r(t)$ satisfies

$$\dot{e}(t) = A_r e(t),\ \lim_{t\to\infty} e(t) = 0 \text{ exponentially}$$

Need of adaptive control

K_1^* and k_2^* depend on A and B unknown.

Adaptive controller for unknown system

$$u(t) = K_1^T(t)x(t) + k_2(t)r(t)$$

$K_1(t)$, $k_2(t)$: adaptive estimates of unknown K_1^*, k_2^*

Closed-loop system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B\left(K_1^T(t)x(t) + k_2(t)r(t)\right)\\ &= A_r x(t) + B_r r(t) + B_r\left(\frac{1}{k_2^*}\tilde{K}_1^T(t)x(t) + \frac{1}{k_2^*}\tilde{k}_2(t)r(t)\right)\end{aligned}$$

$$\tilde{K}_1(t) = K_1(t) - K_1^*,\ \tilde{k}_2(t) = k_2(t) - k_2^*$$

Tracking error equation

$$\dot{e}(t) = A_r e(t) + B_r\left(\frac{1}{k_2^*}\tilde{K}_1^T(t)x(t) + \frac{1}{k_2^*}\tilde{k}_2(t)r(t)\right)$$

Adaptive laws

$$\dot{K}_1(t) = -\mathrm{sign}[k_2^*]\Gamma x(t)e^T(t)PB_r,\ \Gamma = \Gamma^T > 0$$

$$\dot{k}_2(t) = -\mathrm{sign}[k_2^*]\gamma r(t)e^T(t)PB_r,\ \gamma > 0$$

with $K_1(0)$ and $k_2(0)$ being arbitrary, and $P = P^T > 0$

Lyapunov function

$$V(e, \tilde{K}_1, \tilde{k_2}) = e^T Pe + \frac{1}{|k_2^*|}\tilde{K}_1^T\Gamma^{-1}\tilde{K}_1 + \frac{1}{|k_2^*|}\tilde{k}_2^2\gamma^{-1}$$

$$\dot{V} = -e^T(t)Qe(t),\ Q = Q^T > 0$$

System properties

(i) $V(t)$ is bounded, and so are $x(t)$, $K_1(t)$, $k_2(t)$, $u(t)$;

(ii) $\dot{e}(t)$ is bounded, $e(t) \in L^2$, so that $\lim_{t\to\infty} e(t) = 0$.

State Feedback Design for Output Tracking

System model

$$y(s) = C(sI - A)^{-1}Bu(s) = \frac{Z(s)}{P(s)}u(s)$$

$$P(s) = \det(sI - A) = s^n + p_{n-1}s^{n-1} + \cdots + p_1 s + p_0$$

$$Z(s) = k_p(s^{n-n^*} + z_{n-n^*-1}s^{n-n^*-1} + \cdots + z_1 s + z_0)$$

the state variable x is available for measurement.

Reference system

$$y_r(t) = W_r(s)[r](t),\; W_r(s) = \frac{1}{P_r(s)}$$

$P_r(s)$: stable polynomial of degree n^*; $r(t)$: bounded

Design conditions

(A1) $Z(s)$ is a stable polynomial, and

(A2) the system relative degree n^* is known.

Ideal controller for known system

$$u(t) = K_1^{*T} x(t) + k_2^* r(t)$$

Matching conditions

$$\det(sI - A - BK_1^{*T}) = P_r(s)Z(s)/k_p,\ k_2^* = 1/k_p$$

Closed-loop system

$$\dot{x}(t) = (A + BK_1^{*T})x(t) + Bk_2^* r(t),\ y(t) = Cx(t)$$

$$y(s) = \frac{Z(s)}{\det(sI - A - BK_1^{*T})} k_2^* r(s) = W_r(s)r(s)$$

$$\lim_{t \to \infty} (y(t) - y_r(t)) = 0$$

Remarks

(i) guarantee of matching conditions

(ii) need of adaptive control.

Adaptive controller for unknown system

$$u(t) = K_1^T(t)x(t) + k_2(t)r(t)$$

$K_1(t) \in R^n$, $k_2(t) \in R$: adaptive estimates of unknown K_1^*, k_2^*

Closed-loop system

$$\dot{x}(t) \;=\; (A + BK_1^{*T})x(t) + Bk_2^* r(t) + B(\tilde{K}_1^T(t)x(t) + \tilde{k}_r(t)r(t))$$

$$y(t) = Cx(t) = y_r(t) + k_p W_r(s)[(K_1 - K_1^*)^T x + (k_r - k_r^*)r](t)$$

Estimation error

$$\varepsilon(t) = y(t) - y_r(t) + \rho(t)\xi(t)$$

$\rho(t)$: estimate of $\rho^* = k_p$, and

$$\xi(t) = \theta^T(t)\zeta(t) - W_r(s)[\theta^T \omega](t),\; \zeta(t) = W_r(s)[\omega](t)$$

$$\theta(t) = [K_1^T(t), k_2(t)]^T,\; \omega(t) = \left[x^T(t), r(t)\right]^T$$

Adaptive laws

$$\dot{\theta}(t) = -\frac{\Gamma \mathrm{sign}[k_p]\zeta(t)\varepsilon(t)}{1+\zeta^T(t)\zeta(t)+\xi^2(t)},\ \Gamma = \Gamma^T > 0$$

$$\dot{\rho}(t) = -\frac{\gamma\xi(t)\varepsilon(t)}{1+\zeta^T(t)\zeta(t)+\xi^2(t)},\ \gamma > 0$$

Stability properties

$$V(\tilde{\theta},\tilde{\rho}) = \frac{1}{2}(|\rho^*|\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}+\gamma^{-1}\tilde{\rho}^2),\ \dot{V} = -\frac{\varepsilon^2(t)}{z^2(t)}$$

(i) $\theta(t) \in L^\infty$, $\rho(t) \in L^\infty$, $\frac{\varepsilon(t)}{z(t)} \in L^2 \cap L^\infty$, $\dot{\theta}(t) \in L^2 \cap L^\infty$, and $\dot{\rho}(t) \in L^2 \cap L^\infty$, $z^2(t) = 1+\zeta^T(t)\zeta(t)+\xi^2(t)$; and

(ii) all signals in the closed-loop system are bounded, and

$$\lim_{t\to\infty}(y(t)-y_r(t)) = 0,\ \int_0^\infty (y(t)-y_r(t))^2 dt < \infty.$$

Output Feedback Design for Output Tracking

System model

$$y(s) = C(sI - A)^{-1}Bu(s) = \frac{Z(s)}{P(s)}u(s)$$

$$P(s) = \det(sI - A) = s^n + p_{n-1}s^{n-1} + \cdots + p_1 s + p_0$$

$$Z(s) = k_p(s^{n-n^*} + z_{n-n^*-1}s^{n-n^*-1} + \cdots + z_1 s + z_0)$$

only the output variable $y = Cx$ is available for measurement.

Reference system

$$y_r(t) = W_r(s)[r](t),\; W_r(s) = \frac{1}{P_r(s)}$$

$P_r(s)$: stable polynomial of degree n^*; $r(t)$: bounded

Design conditions

(A1) $Z(s)$ is a stable polynomial, and

(A2) the system relative degree n^* is known.

Ideal controller for known system

$$u(t) = \theta_1^{*T}\omega_1(t) + \theta_2^{*T}\omega_2(t) + \theta_{20}^{*}y(t) + \theta_3^{*}r(t)$$

$$\omega_1(t) = \frac{a(s)}{\Lambda(s)}[u](t),\ \omega_2(t) = \frac{a(s)}{\Lambda(s)}[y](t),\ a(s) = [1, s, \cdots, s^{n-2}]^T$$

$\theta_1^*, \theta_2^* \in R^{n-1}$, $\theta_{20}^*, \theta_3^* \in R$, $\Lambda(s)$: stable of degree $n-1$

Matching condition

$$\begin{aligned} &\theta_1^{*T}a(s)P(s) + (\theta_2^{*T}a(s) + \theta_{20}^{*}\Lambda(s))Z(s) \\ =\ &\Lambda(s)(P(s) - \theta_3^{*}Z(s)P_r(s)) \end{aligned}$$

Closed-loop system

$$y(t) = G(s)\left(1 - F_1(s) - F_2(s)G(s) - \theta_{20}^{*}G(s)\right)^{-1}\theta_3^{*}r(s) = W_r(s)r(s)$$

$$\lim_{t\to\infty}(y(t) - y_r(t)) = 0$$

Adaptive controller for unknown system

$$u(t) = \theta_1^T(t)\omega_1(t) + \theta_2^T(t)\omega_2(t) + \theta_{20}(t)y(t) + \theta_3(t)r(t)$$

$\theta_1(t)$, $\theta_2(t)$, $\theta_{20}(t)$, $\theta_3(t)$: adaptive estimates of θ_1^*, θ_2^*, $\theta_{20}^*, \theta_3^*$

Closed-loop system

$$y(t) - y_r(t) = \frac{k_p}{P_r(s)}[(\theta - \theta^*)^T\omega](t)$$

$$\theta(t) = [\theta_1^T(t), \theta_2^T(t), \theta_{20}(t), \theta_3(t)]^T,\ \theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_{20}^*, \theta_3^*]^T$$

$$\omega(t) = [\omega_1^T(t), \omega_2^T(t), y(t), r(t)]^T$$

Estimation error

$$\varepsilon(t) = y(t) - y_r(t) + \rho(t)\xi(t)$$

$\rho(t)$: estimate of $\rho^* = k_p$, and

$$\xi(t) = \theta^T(t)\zeta(t) - \frac{1}{P_r(s)}[\theta^T\omega](t),\ \zeta(t) = \frac{1}{P_r(s)}[\omega](t)$$

Adaptive laws

$$\dot{\theta}(t) = -\frac{\mathrm{sign}[k_p]\Gamma\varepsilon(t)\zeta(t)}{1+\zeta^T(t)\zeta(t)+\xi^2(t)},\ \Gamma = \Gamma^T > 0$$

$$\dot{\rho}(t) = -\frac{\gamma\varepsilon(t)\xi(t)}{1+\zeta^T(t)\zeta(t)+\xi^2(t)},\ \gamma > 0$$

Stability properties

$$V(\tilde{\theta},\tilde{\rho}) = \frac{1}{2}(|\rho^*|\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}+\gamma^{-1}\tilde{\rho}^2),\ \dot{V} = -\frac{\varepsilon^2(t)}{z^2(t)}$$

(i) $\theta(t) \in L^\infty$, $\rho(t) \in L^\infty$, $\frac{\varepsilon(t)}{z(t)} \in L^2 \cap L^\infty$, $\dot{\theta}(t) \in L^2 \cap L^\infty$, and $\dot{\rho}(t) \in L^2 \cap L^\infty$, $z^2(t) = 1+\zeta^T(t)\zeta(t)+\xi^2(t)$; and

(ii) all signals in the closed-loop system are bounded, and

$$\lim_{t\to\infty}(y(t)-y_r(t)) = 0,\ \int_0^\infty (y(t)-y_r(t))^2 dt < \infty.$$

Adaptive Pole Placement Control

- Relaxation of “stable zeros” condition
- Indirect adaptive control
  - (i) estimation of the unknown system parameters
  - (ii) calculation of controller parameters
- Control singularity problem
  - (i) Diophantine equation
  - (ii) parameter projection
- Rapprochement of adaptive control methods
  - (i) adaptive LQ control
  - (ii) indirect MRAC
  - (iii) direct pole placement control
  - (iv) backstepping design

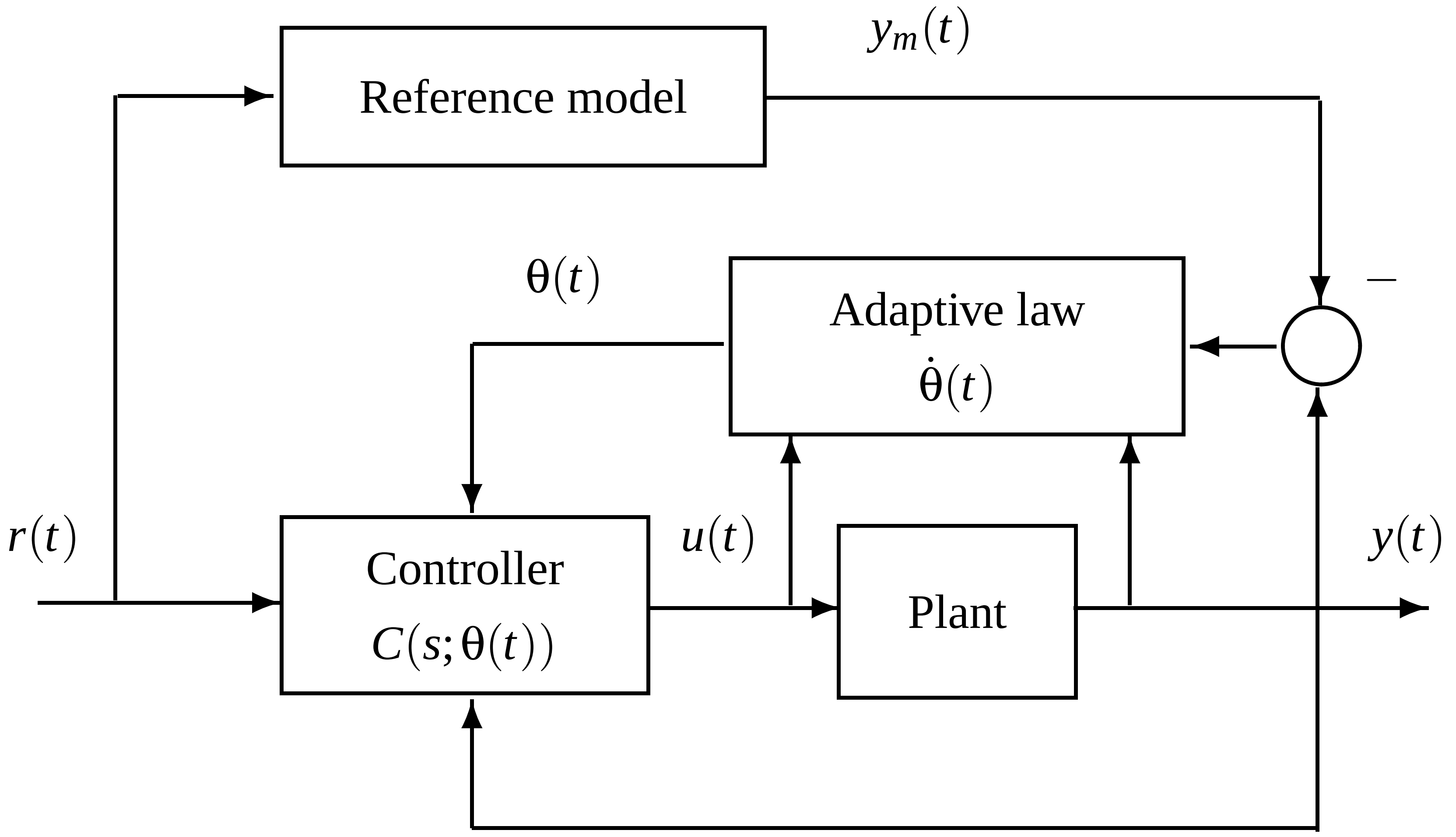
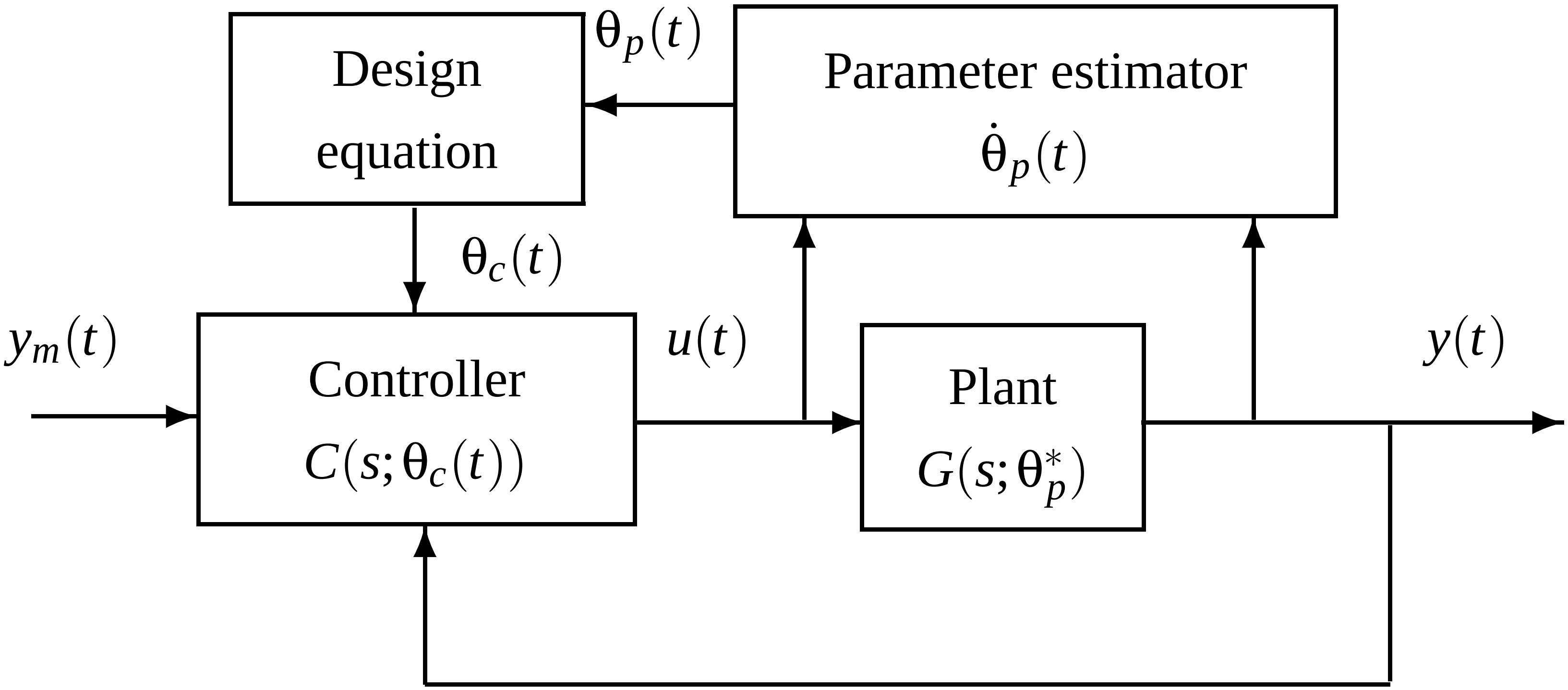


Figure 1: Direct model reference adaptive control system.



Design equation

$\theta_p(t)$

Parameter estimator

$\dot{\theta}_p(t)$

$\theta_c(t)$

$y_m(t)$

Controller

$C(s;\theta_c(t))$

$u(t)$

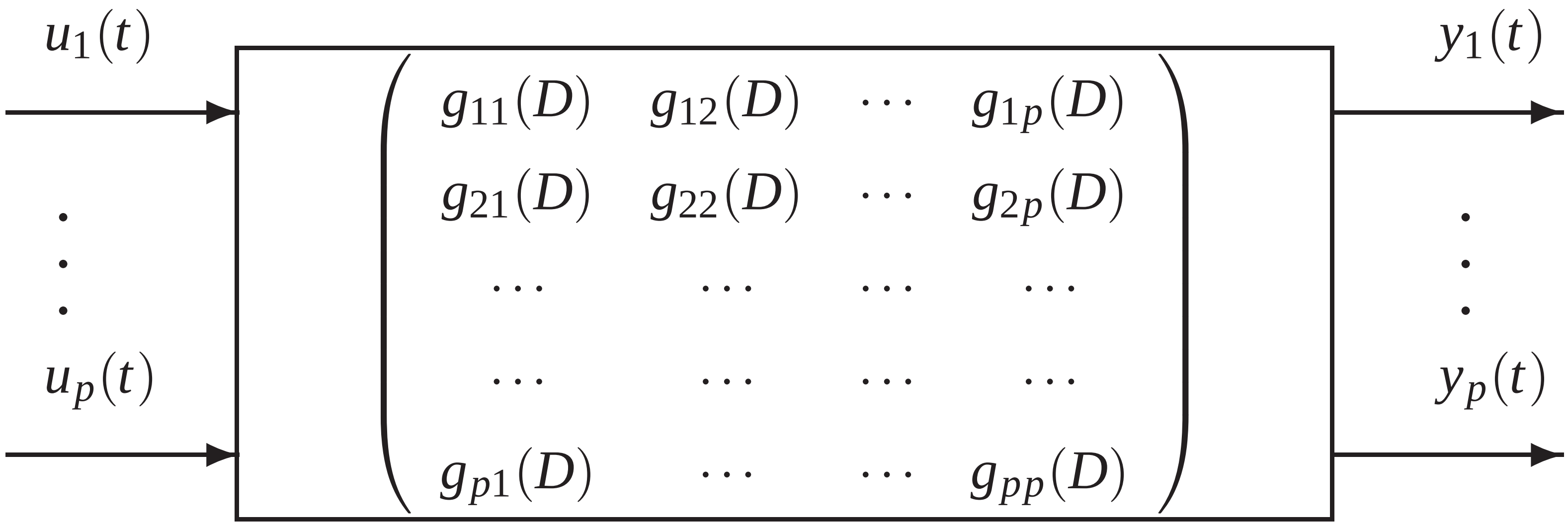
Plant

$G(s;\theta_p^*)$

$y(t)$

Figure 2: Indirect adaptive control system.

Multivariable Adaptive Control



$$\begin{pmatrix} g_{11}(D) & g_{12}(D) & \cdots & g_{1p}(D) \\ g_{21}(D) & g_{22}(D) & \cdots & g_{2p}(D) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ g_{p1}(D) & \cdots & \cdots & g_{pp}(D) \end{pmatrix}$$

- System infinity zero structure (multivariable relative degree)
- System and controller parametrizations
- Dynamics decoupling
- Controller adaptation

Nonlinear Adaptive Control

- System model

$$\dot{x} = f(x) + g(x)u,\ y = h(x),\ x = [x_1, x_2, \ldots, x_n]^T$$

- Feedback linearization
  - diffeomorphism $z = T(x) = [z_1, \ldots, z_\rho, z_{\rho+1}, \ldots, z_n]^T = [\xi^T, \eta^T]^T$
  - system transformation

$$\begin{aligned} \dot{z}_1 &= z_2,\ \dot{z}_2 = z_3, \ldots,\ \dot{z}_{\rho-1} = z_\rho, \\ \dot{z}_\rho &= b(\xi,\eta) + a(\xi,\eta)u,\ \dot{\eta} = q(\xi,\eta) \end{aligned}$$

  - feedback linearizing design

$$u = a^{-1}(\xi,\eta)(v - b(\xi,\eta)) \Rightarrow \dot{z}_1 = z_2,\ \ldots,\ \dot{z}_{\rho-1} = z_\rho,\ \dot{z}_\rho = v$$

  - linear feedback design for v: $v = K_1\xi + K_2\eta + r$

- adaptive control: parametrization of $T(x)$, $a(\xi,\eta)$, $b(\xi,\eta)$
- zero dynamics system $\dot{\eta} = q(\xi,\eta)$

- Backstepping designs

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta^{*T}\varphi_1(x_1),\\ \dot{x}_2 &= \varphi_0(x_1,x_2) + \theta^{*T}\varphi_2(x_1,x_2) + u, \;\; y = x_1\end{aligned}$$

  - objective: tracking of $y_r(t)$ by $y(t)$
  - step 1: introduce $z_1 = x_1 - y_r$, $z_2 = x_2 - \alpha_1$, α_1 to be defined, consider the measure of errors z_1, $\theta - \theta^*$:

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}(\theta-\theta^*)^T\Gamma^{-1}(\theta-\theta^*),\ \Gamma = \Gamma^T > 0$$

$$\dot{V}_1 = z_1(z_2 + \alpha_1 + \theta^T\varphi_1(x_1) - \dot{y}_r) + (\theta-\theta^*)^T\Gamma^{-1}(\dot{\theta} - \Gamma z_1\varphi_1(x_1))$$

choose the stabilizing function

$$\alpha_1 = -c_1 z_1 - \theta^T\varphi_1(x_1) + \dot{y}_r,\ c_1 > 0$$

and the tuning function $\tau_1 = \Gamma z_1 \varphi_1(x_1)$, leading to

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 + (\theta - \theta^*)^T \Gamma^{-1} (\dot{\theta} - \tau_1)$$

- Step 2: consider the full error measure $V_2 = V_1 + \frac{1}{2} z_2^2$, choose the adaptive law for parameter estimate θ:

$$\dot{\theta} = \tau_2 = \tau_1 + \Gamma z_2 \left(\varphi_2 - \frac{\partial \alpha_1}{\partial x_1} \varphi_1 \right)$$

and the stabilizing control

$$\begin{aligned} u(t) \; = \; & -z_1 - \varphi_0(x_1, x_2) - c_2 z_2 + \frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial \theta} \tau_2 + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r \\ & + \frac{\partial \alpha_1}{\partial \dot{y}_r} \ddot{y}_r - \theta^T \left(\varphi_2(x_1, x_2) - \frac{\partial \alpha_1}{\partial x_1} \varphi_1(x_1) \right), \; c_2 > 0 \end{aligned}$$

leading to the desired nonincreasing energy V_2:

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2.$$

- Output feedback designs
  - output-feedback canonical form systems

$$
\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_{01}(y) + \sum_{i=1}^{q} a_i \varphi_{i1}(y),\ \dot{x}_2 = x_3 + \varphi_{02}(y) + \sum_{i=1}^{q} a_i \varphi_{i2}(y),\ \cdots,\\
\dot{x}_{\rho-1} &= x_\rho + \varphi_{0\rho-1}(y) + \sum_{i=1}^{q} a_i \varphi_{i\rho-1}(y),\\
\dot{x}_\rho &= x_{\rho+1} + \varphi_{0\rho}(y) + \sum_{i=1}^{q} a_i \varphi_{i\rho}(y) + b_{n-\rho}\sigma(y)u,\ \cdots,\\
\dot{x}_n &= \varphi_{0n}(y) + \sum_{i=1}^{q} a_i \varphi_{in}(y) + b_0\sigma(y)u,\ \ y = x_1
\end{aligned}
$$

$$\Rightarrow \dot{x} = Ax + \varphi_0(y) + \sum_{i=1}^{q} a_i \varphi_i(y) + b\sigma(y)u,\ y = cx$$

$a_1, \ldots, a_q, b_0, \ldots, b_{n-\rho}$: unknown constant parameters

$\varphi_{ij}, i = 0, \ldots, q,\ j = 1, \ldots, n$, σ: known smooth nonlinear functions.

- state observer: choose $k = [k_1, \ldots, k_n]^T$ to make $A_0 = A - kc$ stable, define the state estimate $\hat{x}$ from

$$\begin{aligned} \hat{x}(t) &= \xi_0 + \sum_{i=1}^{q} a_i \xi_i + \sum_{i=0}^{n-\rho} b_i v_i \\ \dot{\xi}_0 &= A_0 \xi_0 + ky + \varphi_0(y) \\ \dot{\xi}_i &= A_0 \xi_i + \varphi_i(y),\ i = 1, \ldots, q \\ \dot{v}_i &= A_0 v_i + e_{n-i} \sigma(y) u,\ i = 0, 1, \ldots, n - \rho \end{aligned}$$

  leading to the state estimation error $\varepsilon(t) = x(t) - \hat{x}(t)$: $\dot{\varepsilon} = A_0 \varepsilon$

- adaptive backstepping design for a_i, b_j unknown
- minimum phase condition: $B(s) = b_{n-\rho} s^{n-\rho} + \cdots + b_0$ stable

- Nonlinear function approximation based designs
- Adaptive control for nonsmooth nonlinearities

Model-Based versus Approximation-Based: Example

- Aircraft wing rock model

$$\dot{x}_1 = x_2,\ \dot{x}_2 = f_2(x) + d_0 u$$

nonlinearity: $f_2(x) = b_0 + b_1 x_1 + b_2 x_2 + b_3 |x_1| x_2 + b_4 |x_2| x_2 + b_5 x_1^3$

$x_1 = \phi$: roll angle, $x_2 = \dot{\phi}$: roll rate, u: aileron deflection angle

d_0, b_i, $i = 0, 1, \ldots, 5$: unknown parameters

- Reference system

$$\dot{x}_r = A_r x_r + B_r r,\ A_r = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix},\ B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Error system

$$\dot{e} = A_r e + B_r(\Delta(x) + d_0 u - r),\ e = x - x_r$$

$\Delta(x) = b_0 + (b_1 + \omega_n^2) x_1 + (b_2 + 2\zeta\omega_n) x_2 + b_3 |x_1| x_2 + b_4 |x_2| x_2 + b_5 x_1^3$

- Nominal control (leading to $\dot{e} = A_r e$):

$$
\begin{aligned}
u &= -\frac{\Delta(x)}{d_0} + \frac{r}{d_0} \\
&= \theta_0^* + \theta_1^* x_1 + \theta_2^* x_2 + \theta_3^* |x_1| x_2 + \theta_4^* |x_2| x_2 + \theta_5^* x_1^3 + \theta_6^* r
\end{aligned}
$$

- Model-based adaptive control

$$u = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 |x_1| x_2 + \theta_4 |x_2| x_2 + \theta_5 x_1^3 + \theta_6 r$$

$$\dot{\theta} = -\mathrm{sign}[d_0] \Gamma e^T P B_r h(x,r),\ \Gamma = \Gamma^T > 0$$

$$h(x,r) = [1, x_1, x_2, |x_1| x_2, |x_2| x_2, x_1^3, r]^T,\ P A_r + A_r^T P = -Q < 0$$

leading to global stability and tracking properties:

$$V = e^T P e + |d_0| (\theta - \theta)^T \Gamma^{-1} (\theta - \theta^*),\ \dot{V} = -2 e^T Q e$$

$$\Rightarrow x(t) \in L_\infty,\ e(t) \in L_2,\ \lim_{t \to \infty} (x(t) - x_r(t)) = 0$$

for any bounded $r(t)$ and any $x(0)$ and $x_r(0)$.

- Neural network based adaptive control ($r = 0$, $d_0 = 1$)
  - radial basis function based design

$$u = -w^T(t)\psi(x),\ \psi_i = \exp(-\|x - c_i\|^2/s_i^2)$$

$$\dot{w} = \Gamma e^T P B_r \psi(x),\ \hat{\Delta}(x) = w_0^* + \sum_{i=1}^{n} w_i^* \psi_i$$

$$\Delta(x) = \hat{\Delta}(x) + \varepsilon_1(x),\ |\varepsilon_1(x)| < \varepsilon_0,\ \forall x \in D \subset R^2$$

  - single hidden layer NN based design

$$u = -W^T(t)\sigma(V^T(t)\bar{x}),\ \bar{x} = [b_v, x_1, x_2]^T$$

$$\dot{V} = \Gamma_v(\bar{x}e^T P B_r W^T \sigma' - \kappa_v V),\ \dot{W} = \Gamma_w\left((\sigma - \sigma' V^T \bar{x})e^T P B_r - \kappa_w W\right)$$

$$\hat{\Delta}(x) = W^{*T}\sigma(V^{*T}\bar{x}),\ \Delta(x) = \hat{\Delta}(x) + \varepsilon_2(x),\ |\varepsilon_2(x)| < \varepsilon_0,\ \forall x \in D \subset R^2$$

  - closed-loop property: bounded and possibly small $e(t)$
  - issues: (i) no guarantee of $\lim_{t\to\infty} e(t) = 0$

    (ii) size of D, selection of ε_0 and NNs, size of $e(t)$.

Model-Based versus Approximation-Based Control

- Model-based adaptive control
  - diverse control and parameter adaptation algorithms
  - using system structure model information
  - complete parametrization of uncertain system
  - global signal boundedness and *asymptotic tracking*
  - no need of high-gain control
- Universal-approximation based control
  - examples: neural networks, fuzzy logic based designs
  - using reduced system model information
  - approximation region and approximation error
  - signal boundedness
  - reduced tracking error by high-gain control.

Model-Based Control for Aerospace Systems

- Advantages as the main framework control system
  - effective use of system structure information
    “essential modeling information is very often available”
  - choice of well-developed design and analysis methods
  - use of specific controller structures
    “assigning a pilot not just a ‘driver’ to fly an aircraft”
  - characterizable performance specifications
  - guaranteed stability and asymptotic tracking
    “landing an aircraft or positioning a satellite with no error”
- Improvement of model-based control designs
  - combined use of approximation-based design signals
  - development of new designs for specific performance needs.

Parameter Convergence in Adaptive Control

- Convergence of parameter estimates to their nominal values?
- Sufficient but not necessary for stability and tracking
- Persistent excitation for parameter convergence

$$\varepsilon(t) = (\theta(t) - \theta^*)^T \zeta(t),\ \theta^* \in R^{n_\theta}$$

$\zeta(t)$ is persistently exciting if there exist $\delta > 0$ and $\alpha_0 > 0$:

$$\int_{\sigma}^{\sigma+\delta} \zeta(t)\zeta^T(t)\,dt \geq \alpha_0 I,\ \forall\sigma \geq t_0$$

  - adaptive control: $\zeta(t) - \zeta_r(t) \in L^2$, $\zeta_r(t) = H(s)[r](t)$
  - $\zeta(t)$ is PE if $H(j\omega_i)$, $i = 1, 2, \ldots, n_\theta$, are linearly independent (co-primeness of system model and no overparametrization of controller), and $r(t)$ has n_θ frequencies (sufficiently rich).

Adaptive Disturbance Rejection

- System model

$$\dot{x}(t) = Ax(t) + Bu(t) + B_d d_u(t),\ y(t) = Cx(t) + d_y(t)$$

$$d(t) = d_0 + \sum_{j=1}^{q} d_j f_j(t)$$

unknown constants d_0, d_j and *known* bounded signals $f_j(t)$

- State tracking condition: $B_d = \alpha B$
- Output tracking: (A, B, C), (A, B_d, C) of same relative degree
- Sinusoidal disturbances: $f_j(t) = \sin \omega_j t$
  - ω_j known: same relative degree not needed
  - comparison: IMP control uses ω_j, reduced stability region
  - ω_j unknown (so is $f_j(t)$): estimation of ω_j

Robust Adaptive Control

- System model

$$\begin{aligned} y(t) &= G(s)[u](t) + d(t) \\ G(s) &= G_0(s)(1 + \mu\Delta_m(s)) + \mu\Delta_a(s) \end{aligned}$$

- Adaptive control designs for $y(t) = G_0(s)[u](t)$

$$\dot{\theta}(t) = -\Gamma\varepsilon(t)\zeta(t)$$

- Performance robustness with respect to $d(t)$, $\mu\Delta_a(s)$, $\mu\Delta_m(s)$
- Modified adaptive laws for robustness

$$\dot{\theta}(t) = -\Gamma\varepsilon(t)\zeta(t) + f(t)$$

$f(t)$: dead-zone, projection, feedback modifications, etc.

- Parametrization versus robustness

Concluding Remarks

- System uncertainties
  - common in control systems
  - challenges for system performance
- Adaptive control
  - handles system uncertainties effectively
  - ensures desired asymptotic performance
- Adaptive control theory
  - mature with systematic design procedures
  - developing with new challenges
- Adaptive control techniques
  - proved to be useful for many practical control problems
  - promising for new aerospace applications

Our Recent Work

- G. Tao and P. V. Kokotović, *Adaptive Control of Systems with Actuator and Sensor Nonlinearities*, John Wiley & Sons, 1996.
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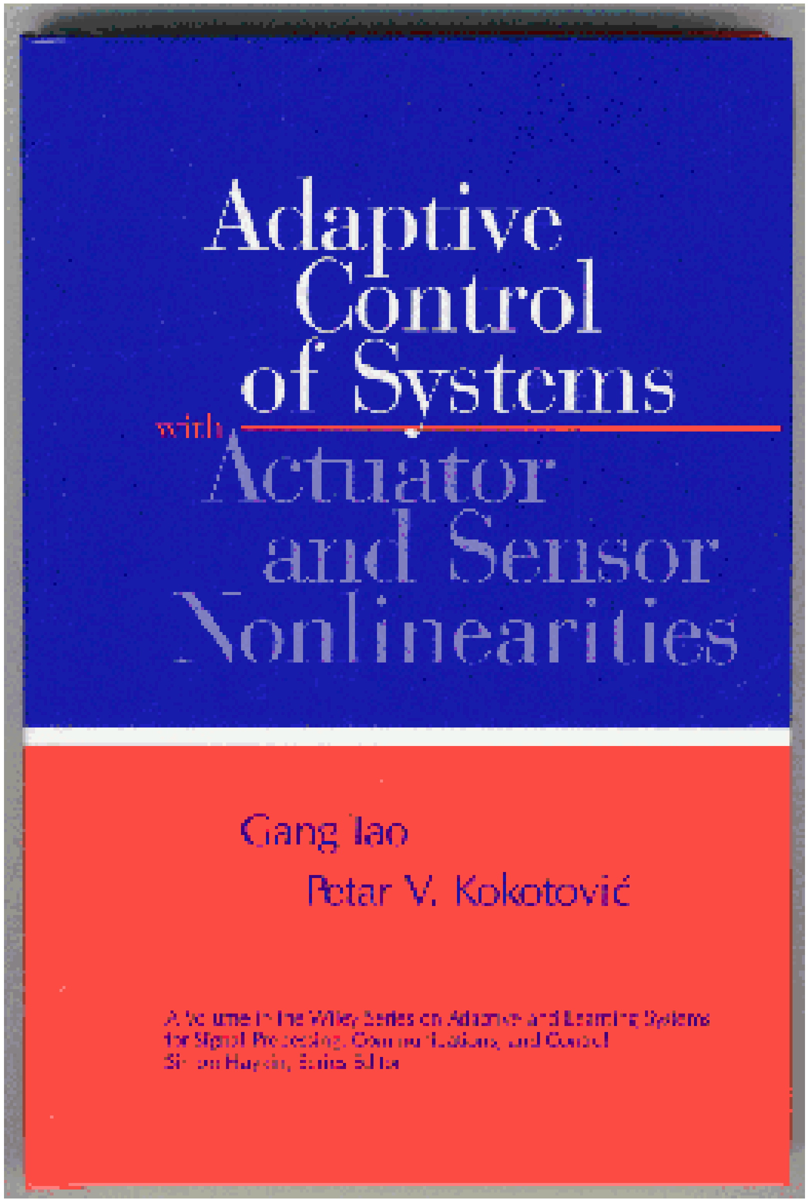

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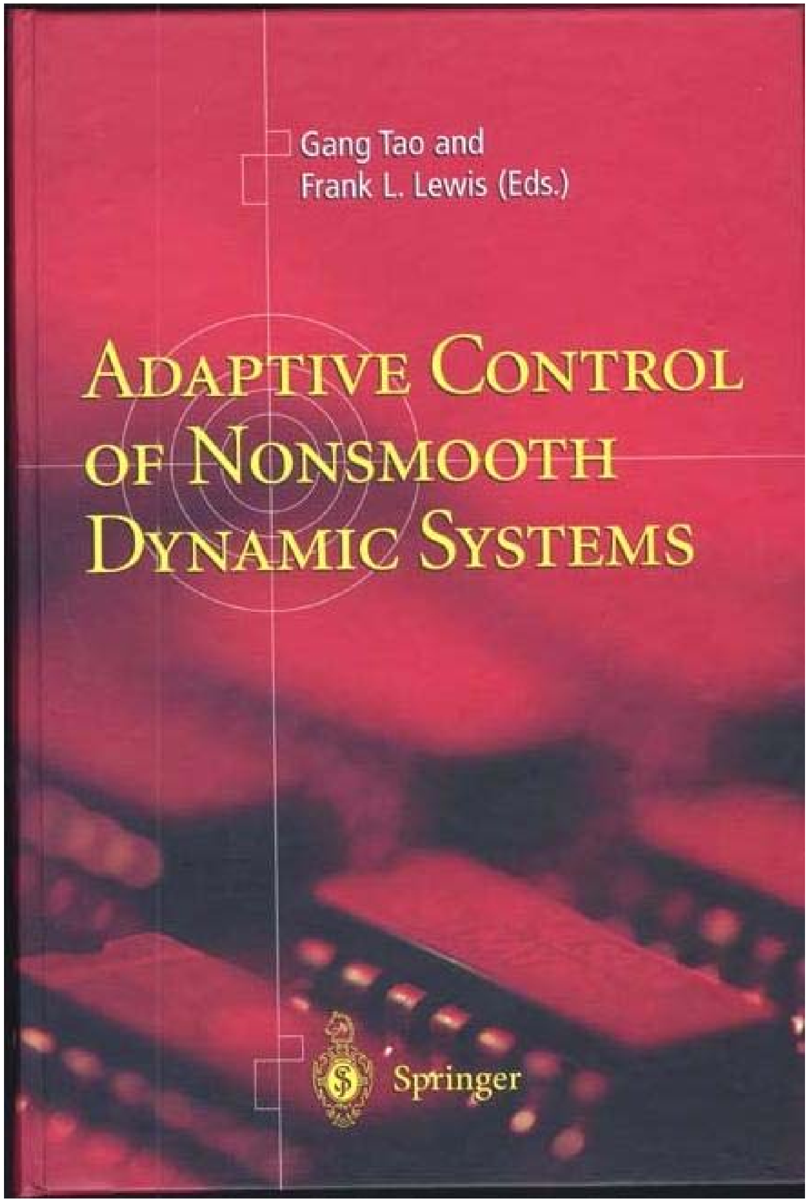

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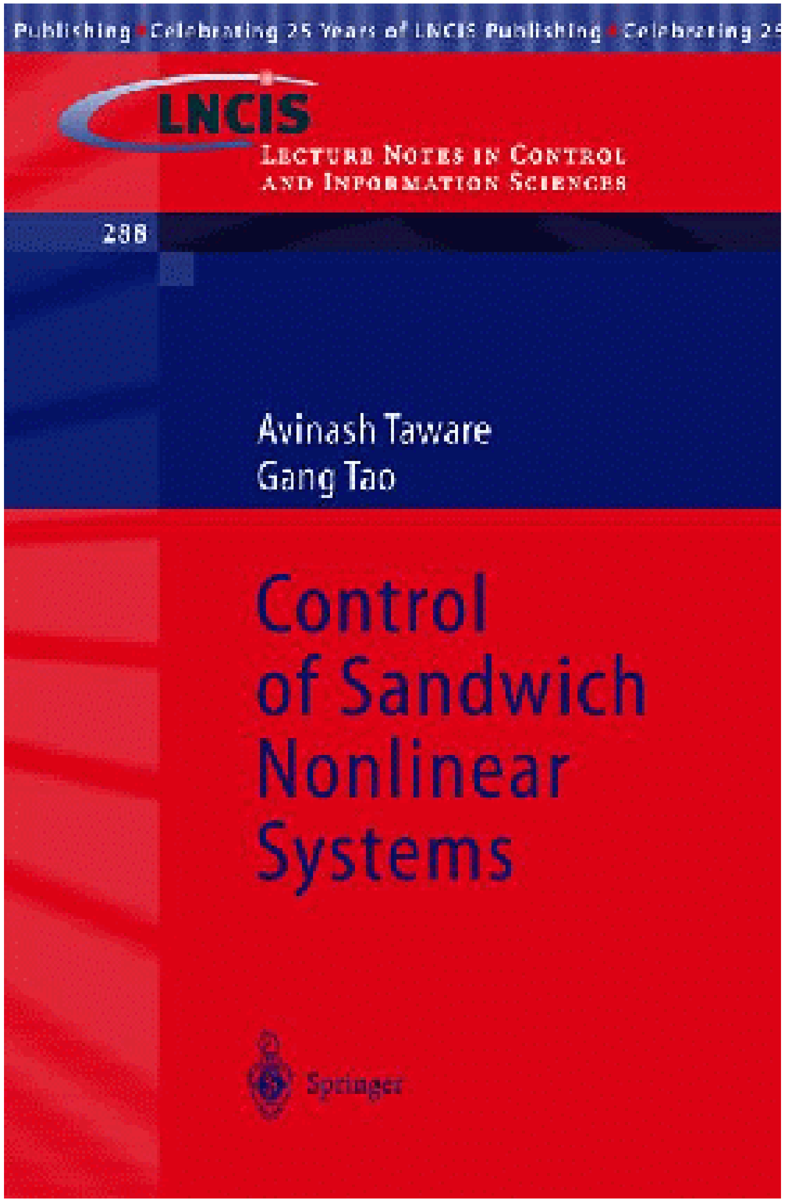

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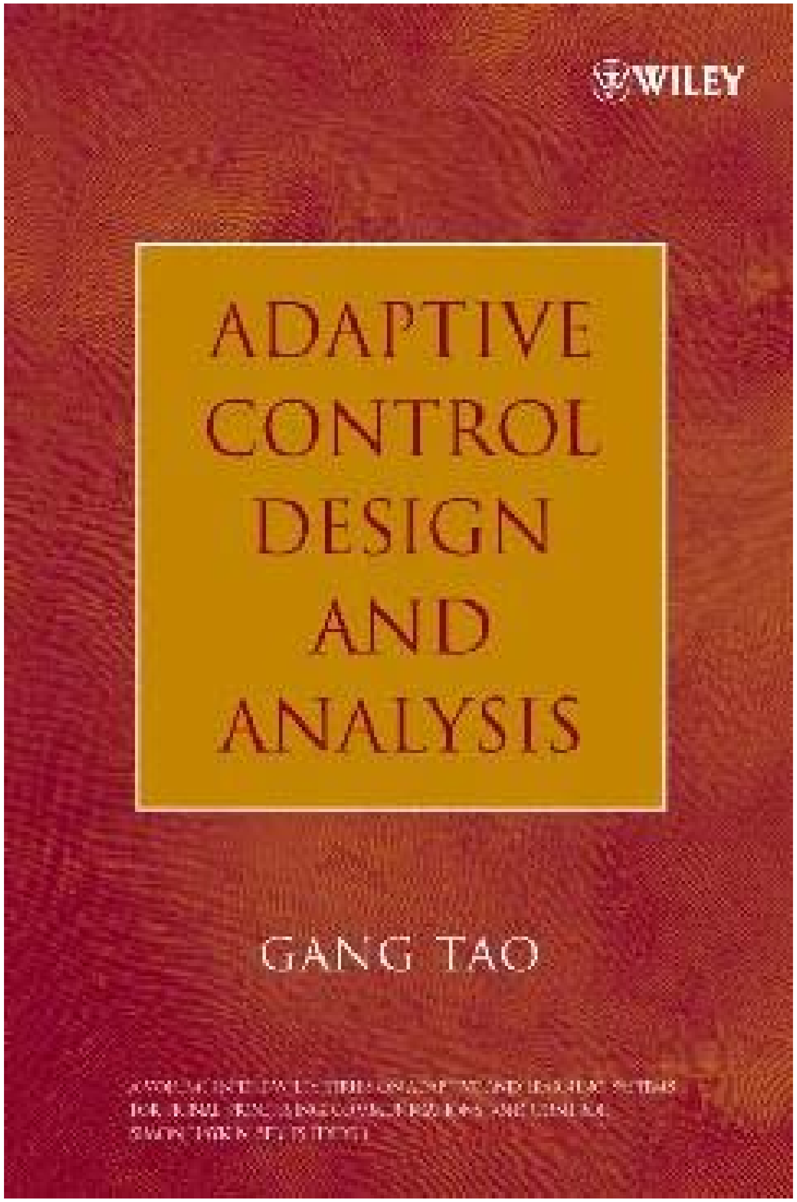

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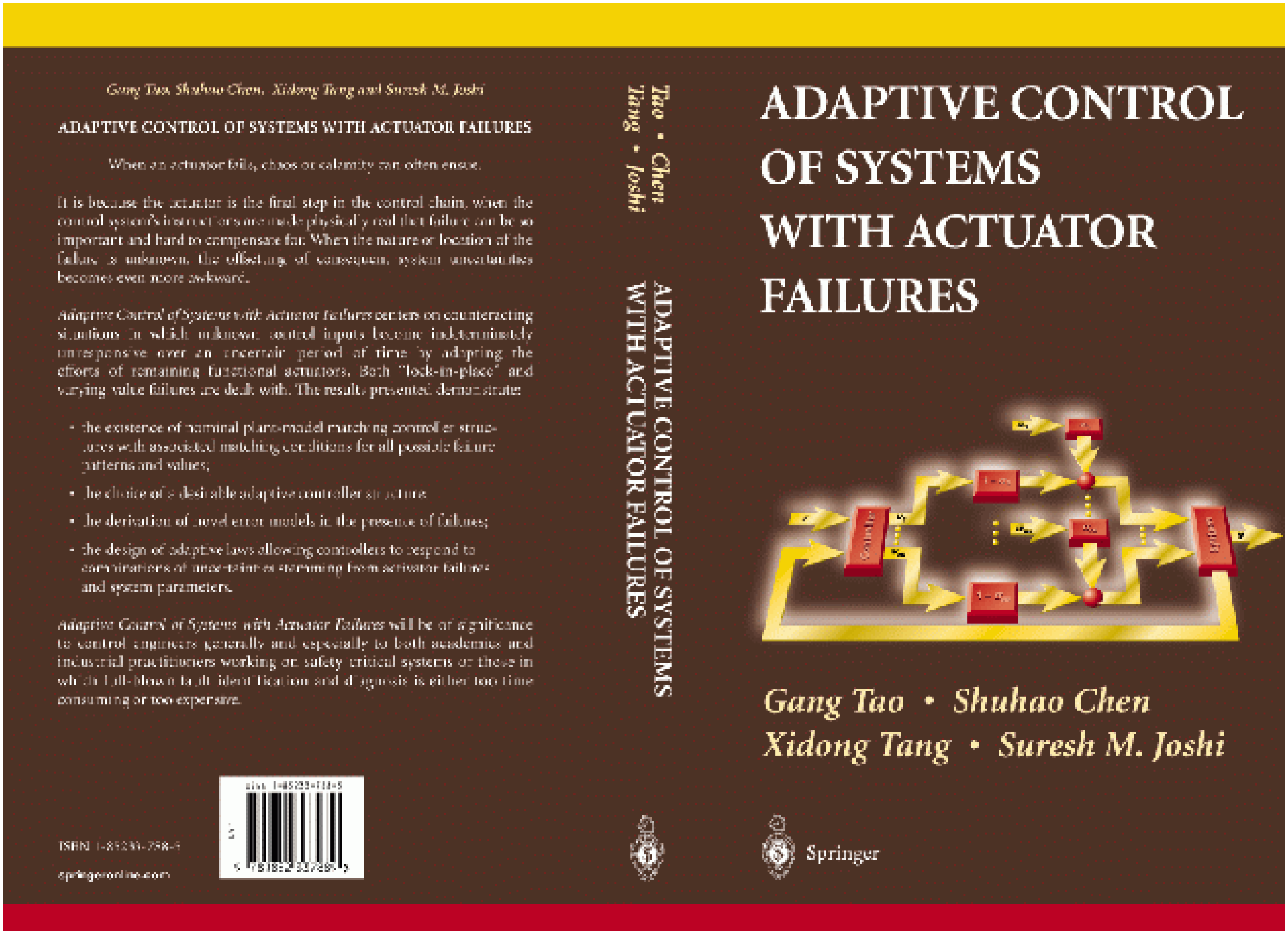

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ADAPTIVE CONTROL OF SYSTEMS WITH ACTUATOR FAILURES

When an actuator fails, chaos or calamity can often ensue.

It is because the actuator is the final step in the control chain, when the control system's instructions are made physically real that failure can be so important and hard to compensate for. When the nature or location of the failure is unknown, the offsetting of consequent system uncertainties becomes even more awkward.

Adaptive Control of Systems with Actuator Failures centers on counteracting situations in which unknown control inputs become indeterminately unresponsive over an uncertain period of time by adapting the efforts of remaining functional actuators. Both "lock-in-place" and varying value failures are dealt with. The results presented demonstrate:

- the existence of nominal plant-model matching controller structures with associated matching conditions for all possible failure patterns and values;
- the choice of a desirable adaptive controller structure;
- the derivation of novel error models in the presence of failures;
- the design of adaptive laws allowing controllers to respond to combinations of uncertainties stemming from actuator failures and system parameters.

Adaptive Control of Systems with Actuator Failures will be of significance to control engineers generally and especially to both academics and industrial practitioners working on safety critical systems or those in which full-blown fault identification and diagnosis is either too time consuming or too expensive.

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- Adaptive Inverse Control for Synthetic Jet Actuators
- Adaptive Actuator Failure Compensation
- Design for Linearized Longitudinal B737 Model
- Design for Linearized Lateral B737 Model
- Open Research Problems

Questions?

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